COURSE HANDOUT

Course Code	ACSC13
Course Name	Design and Analysis of Algorithms
Class / Semester	IV SEM
Section	A-SECTION
Name of the Department	CSE-CYBER SECURITY
Employee ID	IARE11023
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Topic Covered	Asymptotic Notations- Big, Omega, and Theta Notations examples
Course Outcome/s	Compute the time complexity with asymptotic notations
Handout Number	13
Date	12 April, 2023

Content about topic covered: Big Oh, Omega, and Theta Notations

Big O notation: The function f(n) = O(g(n)) iff (if and only if) there exist positive constants c and n_0 such that $f(n) \leq c^* g(n)$ for all n, $n \geq n_0$.

The statement f(n) = O(g(n)) states only that g(n) is an **upper bound** on the value of f(n) for all n, $n \geq n_0$.

Eg:

1. $3n+2 = O(n)$ $\quad 3n+2 <= 4n$ $\quad \forall\ n \geq 2$
2. $100n+6 = O(n)$ $\quad 100n+6 <= 101n$ $\quad \forall\ n \geq 6$
3. $10n^2+4n+2 = O(n^2)$ $\quad 10n^2+4n+2 <= 11n^2$ $\quad \forall\ n \geq 5$
4. $6*2^n+n^2 = O(2^n)$ $\quad 6*2^n+n^2 <= 7*2^n$ $\quad \forall\ n \geq 4$

Note: O(1) → Constant line
O(n) → Linear
$O(n^2)$ → Quadratic
$O(n^3)$ → Cubic
$O(2^n)$ → Exponential

Omega notation (Ω): The function f(n) = Ω(g(n)) iff (if and only if) there exist positive constants c and n_0 such that f(n) ≥ c* g(n) for all n, n ≥ n_0.

The statement f(n) = Ω(g(n)) states only that g(n) is a **lower bound** on the value of f(n) for all n, n ≥ n_0.

Eg:

1. $3n+2 = O(n)$ $3n+2 >= 3n$ $\forall\ n \geq 1$
2. $100n +6 = O(n)$ $100n +6 >= 100n$ $\forall\ n \geq 1$
3. $10n^2+4n+2 = O(n^2)$ $10n^2+4n+2 \geq 10n^2$ $\forall\ n \geq 1$

Theta notation (θ): The function f(n) = θ(g(n)) iff (if and only if) there exist positive constants c_1, c_2 and n_0 such that c_1* g(n) ≤ f(n) ≤ c_2* g(n) for all n, n ≥ n_0.

The statement f(n) = θ(g(n)) states only that g(n) is an **both an upper bound and lower bound** on the value of f(n) for all n, n ≥ n_0.

Eg: $3n + 2 = \theta(n)$ $3n \leq 3n+2 \leq 4n$ $\forall\ n \geq 1$

Little Oh notation: The function f(n) = o(g(n) iff

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right)^{\square} = 0$$

Eg: $3n+2 = o(n^2)$

$$\lim_{n\to\infty}\left(\frac{3n+2)}{n^2}\right)^{\square}$$

→

$$\lim_{n\to\infty}\left(\frac{3}{n}+\frac{2)}{n^2}\right)^{\square} = 0$$